

User-Base Station Association in HetSNets: Complexity and Efficient Algorithms

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Abstract—This work considers the problem of user association to small-cell base stations (SBSs) in a heterogeneous and small-cell network (HetSNet). Two optimization problems are investigated, which are maximizing the set of associated users to the SBSs (the unweighted problem) and maximizing the set of weighted associated users to the SBSs (the weighted problem), under signal-to-interference-plus-noise ratio constraints. Both problems are formulated as linear integer programs. The weighted problem is known to be NP-hard and, in this paper, the unweighted problem is proved to be NP-hard as well. Therefore, this paper develops two heuristic polynomial-time algorithms to solve both problems. The computational complexity of the proposed algorithms is evaluated and is shown to be far more efficient than the complexity of the optimal brute-force (BF) algorithm. Moreover, the paper benchmarks the performance of the proposed algorithms against the BF algorithm, the branch-and-bound CPLEX-based algorithm and state-of-the-art algorithms, through numerical simulations. The results demonstrate the close-to-optimal performance of the proposed algorithms. They also show that the weighted problem can be solved to provide solutions that are fair between users or to balance the load among SBSs.

Index Terms—User-BS association, HetSNets, Heuristic algorithm, Brute-force, Branch-and-bound, NP-hard, Fairness.

I. INTRODUCTION

A. Motivation and Research questions

In the last decade, mobile cellular networks have become popular among data users, which has led to a demand for increased capacity. In addition, cellular networks are becoming the main provider of voice and data services with high mobility even though the wireless local area networks (WLANs) can provide higher and less expensive data rates with relatively restricted mobility [1]. In order for cellular networks to be competitive with WLANs, resources must be adequately allocated to provide higher performance and better satisfy the users. To this end, small-cell base stations (SBSs) are introduced to form heterogeneous and small-cell network (HetSNet) [2]. SBSs can provide increase in capacity at low capital expenditure [2]. They have low-power and small-range, and are characterized by their unplanned and plug-and-play features [2], [3]. Despite the gains, the dense deployment of SBSs raises many challenges in HetSNets as the number of SBSs in such networks can exceed the number of users [4]. Such densely deployment of SBSs has made the user-BS association a key challenge. Furthermore, HetSNets are interference-limited and hence the

co-channel interference among SBSs and between SBSs and macro-cell BSs (MBSs) is a critical issue, which needs to be better managed. It is clear that the user-BS association directly affects the interference. However, the basic user-BS association, which pairs the users to the SBSs that has the strongest signal has a low overall throughput because of a poor management of the interference [5]. In this paper, we are interested in finding a user-BS association that increases the number of associated users in one time-slot, such that the signal-to-interference-plus-noise ratio (SINR) of the associated users is guaranteed.

B. Related works

Related work can be broadly divided into two groups. Works on distributed or centralized solutions of the user-BS association problem in HetSNets [6–16] and works on the link activation problem [?], [17], [18]. The link activation problem can be formulated as: given a set of wireless links and their corresponding weights, how to maximize the size of the weighted set of activated links in one time-slot such that the per-link SINR constraint is satisfied [17].

On the one hand, in [6], the authors study the resource allocation in HetSNets as a joint optimization problem of channel allocation, user-BS association, beam-forming and power control. It is solved using an iterative heuristic algorithm based on convex approximations. The work shows that the relaxation of the combinatorial problem to a continuous one is optimal. However, the proof lacks of generality as it depends on the problem formulation. Moreover, the proposed algorithm has high complexity. In [7], the joint power allocation and user-BS association is modeled as a combinatorial optimization problem and solved optimally based on Bender's decomposition. Heuristic algorithms are also proposed. However, both proposed optimal method and heuristic algorithm have high complexity. The work in [8] studies the user-BS association problem for fairness and load balancing (see [14–16] for more papers on fairness and load balancing). It solves the user-BS association problem by relaxation and rounding techniques which remove the combinatorial nature of the problem. Reference [9] solves the user-BS association problem based on a pricing scheme. The authors propose a dual coordinate descent method to solve the problem. They extend the problem to the multiple-input-multiple-output case and optimize the beam-forming variables. The optimization model is very similar to the one in [8]. The main difference with our work is that there are SINR constraints. In [8], [9], multiple users have to be

associated with one BS and all BSs have to be associated in the end of the association scheme which makes the optimization problem simpler. In [10] the joint problem of power control and user-BS association is studied and modeled as a max-min fairness problem. First, the authors study the problem of maximizing the minimum SINR subject to power constraints and the association between the users and the SBSs. Second, they consider the additional constraints of one-to-one matching and of a minimum SINR guarantee. The first problem is shown to be NP-hard and the authors propose a two-stage fixed-point algorithm to solve it. The one-to-one matching problem is shown to be polynomial-time solvable and the authors propose an auction-based algorithm to solve it. In the one-to-one matching problem, the critical assumption made is that the number of users is equal to the number of SBSs and all of them have to be associated (there is no maximization of the number of users) and therefore the authors reduce the problem to an assignment problem. (We discuss the main differences of the user-BS association problem considered in this work and the classical one-tone matching problem in Section IV.) Other interesting works are [11], [12], and [13] where the user-BS association problem is solved jointly either with interference coordination or resource allocation or power control, respectively.

On the other hand, the seminal work of Goussevskaia *et al.* [17] shows that the link activation problem, *one-slot scheduling*, is NP-hard. Note that there is some similarity between [17] and our work. In fact, our problem is somehow equivalent to the one-slot scheduling where the links are not established yet and they have no weights. Anyhow, in [17], the NP-hardness of the unweighted *one-slot scheduling* is not investigated. Moreover, it is important to note that the NP-hardness of the weighted problem does not imply the NP-hardness of the unweighted one [19]. In [?], [18], approximation algorithms and game theoretic distributed solutions are provided in order to solve the joint problem of unweighted *one-slot scheduling* and power allocation under geometric SINR constraints.

To the best of our knowledge, there is no NP-hardness studies of the user-BS association problem in the case of fixed transmit power where the objective is to maximize the set of associated users subject to the SINR constraints. Previous work have focused on simplified assumptions using relaxation techniques which remove the combinatorial nature of the user-BS association problem. Also, the interference constraints are often greatly simplified using graph-based models instead of SINR constraints. Moreover, they do not study the fairness and/or load balancing of the user-BS association problem which is an important aspect in wireless communications. Consequently, in this paper, we study the user-BS association problem under SINR constraints in HetSNet and we prove that it is an NP-hard problem and we also study the fairness and load balancing. The system performance metrics are throughput and fairness/load balancing. Throughput is defined as the number of users that are successfully associated to the SBSs and fairness is measured by the number of times a user is associated to the SBSs.

C. Contributions

This paper investigates two problems of unweighted and weighted user-BS association in an open access HetSNet. We define the unweighted (resp. the weighted) problem as follows: given a set of small-cell users (SUs) and a macro-cell user (MU), a set of SBSs, one MBS, a QoS lower bound and channel gains between every pair of user-BS (resp. a weight for every SU or SBS), the question is to find a set of one-to-one association between the SUs and the SBSs with maximum cardinality (resp. with maximum weighted cardinality) such that the SINR of the SUs and of the MU are greater than the QoS lower bound.

The main contributions of this paper are as follow:

- 1) We prove that the unweighted user-BS association problem is NP-hard.
- 2) We develop efficient heuristic algorithms to solve both unweighted and weighted user-BS association problems.
- 3) We compare the developed algorithms against the brute-force (BF) optimal algorithm, the branch-and-bound (B&B) CPLEX-based algorithm, a standard user-BS association algorithm called max-SINR [5], [9] and a benchmark algorithm recently proposed in [10].
- 4) We evaluate the time complexity of the proposed algorithms and the complexity of the BF algorithm. The complexity of the proposed algorithms is shown to be polynomial in time.

D. Organization

The rest of the paper is organized as follows. Section II discusses the system model. The problem is formulated in Section III. Section IV provides the proof of the NP-hardness of the unweighted user-BS association problem. Next, Section V presents the BF and the B&B CPLEX-based optimal solutions. Section VI presents heuristic algorithms for the user-BS association for both unweighted and weighted problems, and analyzes the complexity of the algorithms. Section VII provides simulation results to compare the algorithms and Section VIII concludes the paper.

II. SYSTEM MODEL

This paper considers the down-link transmission where all BSs transmit over a common frequency band, i.e., frequency reuse one is assumed. The network comprises a MBS, a macro-cell user (MU), several SBSs, and several small-cell users (SUs). We denote by $\mathcal{K} \stackrel{\text{def}}{=} \{1, \dots, K\}$ the set of SUs and by $\mathcal{N} \stackrel{\text{def}}{=} \{1, \dots, N\}$ the set of SBSs. For brevity, a SU and a SBS are denoted by k and n , respectively. The MBS and the MU are denoted by the index 0. The MBS is located in the center of the cell which is modeled as a circle of radius R . SBSs, MU, and SUs are randomly located in this cell following independent two dimensional uniform distributions. An example of the system model is given in Fig. 1.

The wireless channel model includes path loss and Rayleigh fading. The channel between k and n is represented by $g'_{kn} \stackrel{\text{def}}{=} h_{kn} \sqrt{(d_0/d_{kn})^\alpha}$, where α is the path loss coefficient, d_{kn} is the distance between k and n , d_0 is a reference distance

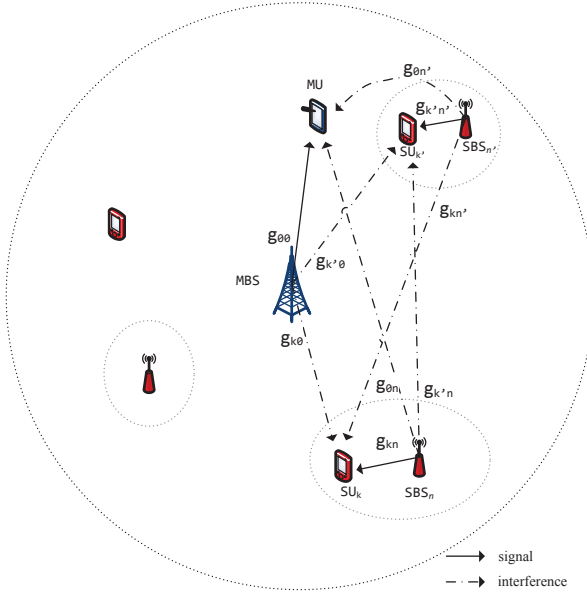


Fig. 1. System Model

at which the reference path loss is calculated (the reader can find more details in [20]), and h_{kn} is the small-scale fading modeled as a zero-mean, complex Gaussian random variable with unit variance. Throughout the rest of the paper, we denote the channel gain by $g_{kn} \stackrel{\text{def}}{=} |g'_{kn}|^2$. In this paper, each transceiver is equipped with a single antenna and one SU can be associated with one and only one SBS.

For the mathematical formulation of the user-BS association problem, the binary variable x_{kn} is defined as follows, for all $k \in \mathcal{K}$ and for all $n \in \mathcal{N}$:

$$x_{kn} \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } k \text{ is associated to } n \\ 0 & \text{otherwise.} \end{cases}$$

The association vector \mathbf{x} , which represents the user-BS association solution, is defined as $\mathbf{x} \stackrel{\text{def}}{=} [x_{11}, \dots, x_{1N} \dots x_{K1}, \dots, x_{KN}]^T \in \{0, 1\}^{K \cdot N}$. Note that one SU can be associated with at most one SBS and one SBS can be associated with at most one SU and hence we have the following one-to-one association constraint on the vector \mathbf{x} : $\sum_{k \in \mathcal{K}} x_{kn} \leq 1$ and $\sum_{n \in \mathcal{N}} x_{kn} \leq 1$. This one-to-one assumption is reasonable since we assume that transmissions between SBSs and SUs occur during single time-slot, over a common frequency and with single-input-single-output (SISO) network. This paper considers that when a SU is associated to a SBS in a given time-slot, it means that the SBS transmits to this SU in this time-slot. In this context, transmission and association can be used interchangeably.

If an SBS is able to transmit to more than one SU in the same time slot (for example, multi-antenna SBS), the proposed algorithms can be adapted by treating the SBS as multiple ones having the same channel characteristics.

The transmit power is normalized by the receiver noise power and the reference distance d_0 . The SBSs transmit at constant SNR of γ . Although this assumption is a simplifi-

cation to render the analysis tractable, it has been shown that constant transmit power method is useful in practice [21]. The MBS transmits to its MU at constant SNR of γ_0 . Then, the received SINR at k from n can be written as follows:

$$\text{SINR}_{kn}(\mathbf{x}) \stackrel{\text{def}}{=} \frac{\gamma g_{kn} x_{kn}}{1 + \gamma_0 g_{k0} + \sum_{k' \in \mathcal{K}'} \sum_{n' \in \mathcal{N}'} \gamma g_{kn'} x_{k'n'}}, \quad (1)$$

where $\mathcal{K}' = \mathcal{K} \setminus \{k\}$ and $\mathcal{N}' = \mathcal{N} \setminus \{n\}$.

The SINR at the MU is given by:

$$\text{SINR}_0(\mathbf{x}) \stackrel{\text{def}}{=} \frac{\gamma_0 g_{00}}{1 + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \gamma g_{0n} x_{kn}}. \quad (2)$$

In the above formulation of SINR in (1) and (2), the interference comes from all associated SBSs. This is indicated, in the denominator of equations (1) and (2), by the summation over the channel gains g_{kn} multiplied by the decision variables x_{kn} . Therefore, the interference occurs only when $x_{kn} = 1$.

The minimum required SINR threshold at any SU and at the MU are denoted by β and β_0 , respectively. A user-BS association is feasible if and only if it meets the SINR threshold of the associated SUs and of the MU and if it satisfies the one-to-one association.

III. PROBLEM FORMULATION

A. Unweighted User-BS Association

This section formulates the unweighted user-BS association problem (the unweighted problem). The objective is to maximize the total number of associated SUs in the network subject to the constraints of the received SINR thresholds of the SUs and of the MU.

The problem can be formulated as follows:

$$\underset{\mathbf{x}}{\text{maximize}} \quad \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} x_{kn} \quad (3a)$$

$$\text{subject to} \quad \sum_{k \in \mathcal{K}} x_{kn} \leq 1, \quad \forall n \in \mathcal{N}, \quad (3b)$$

$$\sum_{n \in \mathcal{N}} x_{kn} \leq 1, \quad \forall k \in \mathcal{K}, \quad (3c)$$

$$\text{SINR}_{kn}(\mathbf{x}) \geq \beta x_{kn}, \quad \forall k \in \mathcal{K}, \quad \forall n \in \mathcal{N}, \quad (3d)$$

$$\text{SINR}_0(\mathbf{x}) \geq \beta_0, \quad (3e)$$

$$x_{kn} \in \{0, 1\}, \quad \forall k \in \mathcal{K}, \quad \forall n \in \mathcal{N}. \quad (3f)$$

Constraint (3b) ensures that a SBS associates to one SU whereas constraint (3c) ensures that a SU is associated with one SBS. Constraint (3d) guarantees that a SU associated with a SBS must have an SINR above the threshold β . To ensure the SINR threshold β_0 of the MU, constraint (3e) is introduced. Finally, constraint (3f) ensures that the association variable x_{kn} is Boolean.

Problem (3) can be written in matrix notation. Note that constraint (3d) is nonlinear due to the βx_{kn} term on the right-hand side and the $x_{k'n'}$ in the denominator of the left-hand side. The x_{kn} term dictates that the SINR threshold β

is met only if k is associated to n . If it is not associated (i.e., $x_{kn} = 0$), then the SINR threshold drops to zero and the constraint is satisfied with equality. Using the bigM technique [22], constraint (3d) can be rewritten as below:

$$\frac{\gamma g_{kn} x_{kn} + M(1 - x_{kn})}{1 + \gamma_0 g_{k0} + \sum_{k' \in \mathcal{K}'} \sum_{n' \in \mathcal{N}'} \gamma g_{kn'} x_{k'n'}} \geq \beta, \quad (4)$$

where M is a sufficiently large number so that when $x_{kn} = 0$, constraint (3d) is not violated and on the other hand if $x_{kn} = 1$, the term $M(1 - x_{kn})$ is zero and therefore has no effect. This technique is well known in linear programming. It adds “artificial” variables to the original problem in order to find a feasible solution [22].

The value of M must satisfy the following for all $k \in \mathcal{K}$ and for all $n \in \mathcal{N}$:

$$M \geq \beta + \beta \gamma_0 g_{k0} + \sum_{k' \in \mathcal{K}'} \sum_{n' \in \mathcal{N}'} \beta \gamma g_{kn'} x_{k'n'}. \quad (5)$$

Note that M depends on k , n , and \mathbf{x} . Without loss of generality, we take the highest value of M denoted by M^* :

$$M^* \stackrel{\text{def}}{=} \max_{k, n, \mathbf{x}} \left(\beta + \beta \gamma_0 g_{k0} + \sum_{k' \in \mathcal{K}'} \sum_{n' \in \mathcal{N}'} \beta \gamma g_{kn'} x_{k'n'} \right), \quad (6)$$

Hence, there exists $k^* \in \mathcal{K}$ and there exists $n^* \in \mathcal{N}$ such that equation (6) is satisfied. Then,

$$M^* \stackrel{\text{def}}{=} \beta + \beta \gamma_0 g_{k^*0} + (K - 1) \beta \gamma \sum_{n' \in \mathcal{N}'} g_{k^*n'}, \quad (7)$$

where $\mathcal{N}^* = \mathcal{N} \setminus \{n^*\}$.

Using the previous value of M^* and rearranging the terms, equation (4) is equivalent to:

$$\begin{aligned} (\gamma g_{kn} - M^*) x_{kn} + M^* &\geq \beta + \gamma_0 g_{k0} \beta + \sum_{k' \in \mathcal{K}'} \sum_{n' \in \mathcal{N}'} \gamma \beta g_{kn'} x_{k'n'} \\ &\Leftrightarrow \\ \frac{M^* - \gamma g_{kn}}{M^* - \beta - \beta \gamma_0 g_{k0}} x_{kn} + \sum_{k' \in \mathcal{K}'} \sum_{n' \in \mathcal{N}'} \frac{\gamma \beta g_{kn'}}{M^* - \beta - \beta \gamma_0 g_{k0}} x_{k'n'} &\leq 1. \end{aligned} \quad (8)$$

Also, constraint (3e) can be rewritten as follows:

$$\frac{\gamma_0 g_{00}}{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \gamma g_{0n} x_{kn} + 1} \geq \beta_0 \Leftrightarrow \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \frac{\gamma g_{0n} \beta_0}{\gamma_0 g_{00} - \beta_0} x_{kn} \leq 1. \quad (9)$$

With the above modifications, the unweighted user-BS association problem can be rewritten, in matrix form as follows:

$$\underset{\mathbf{x}}{\text{maximize}} \quad \mathbf{1}^T \mathbf{x} \quad (10a)$$

$$\text{subject to} \quad \mathbf{A} \mathbf{x} \leq \mathbf{1}, \quad (10b)$$

$$\mathbf{x} \in \{0, 1\}^q. \quad (10c)$$

where $\mathbf{1}$ is the unitary vector of size $1 \times q$ and $\mathbf{A} \in \mathbb{R}^{p \times q}$ is the matrix of sizes $p = K + N + K \cdot N + 1$ and $q = K \cdot N$. The matrix \mathbf{A} is defined by $\mathbf{A} = [a_{ij}]$ where a_{ij} can be calculated from (3b), (3c), (8), and (9).

B. Weighted User-BS Association

This section introduces the more general problem of weighted user-BS association (the weighted problem) where each k or n in the network is prioritized by a weight. The problem is to maximize the number of weighted associated SUs subject to the constraints of the received SINR thresholds of the users. The weights can add a degree of fairness to the users or balance the traffic load between the SBSs.

The problem can be formulated by defining a weight vector \mathbf{w} which will be explained mathematically in the sequel. Thus, the weighted user-BS association problem is given below:

$$\underset{\mathbf{x}}{\text{maximize}} \quad \mathbf{w}^T \mathbf{x} \quad (11a)$$

$$\text{subject to} \quad \mathbf{A} \mathbf{x} \leq \mathbf{1}, \quad (11b)$$

$$\mathbf{x} \in \{0, 1\}^q. \quad (11c)$$

Problem (11) is NP-hard [17]. The objective function in (11a) is a linear combination of an association vector \mathbf{x} and a weight vector \mathbf{w} . When the vector \mathbf{w} is set to one, the objective function in (3a) is obtained and hence the unweighted problem is given as in (10).

Weights Design: The weights can be designed based on fair rate or fair time allocation [23]. Since the problem involves user-BS association, we choose the fair time allocation. First, this paper considers the fairness between SUs and second, the weights are designed to provide fairness between SBSs. The fair time allocation between SBSs is also an important aspect and can be seen as a load balancing algorithm.

Every k (resp. n) is associated with a weight $w_k(t)$ (resp. $w_n(t)$) at time-slot t which is, by definition, the reciprocal of number of times k (resp. n) is associated during the previous period of T time-slots, where T is called the window size. Without loss of generality, we assume that the instant time t is at least T , i.e., $t \geq T$. In other words, the weights are initialized for $t \leq T$. To ensure fairness between the SUs, the weights are calculated for every user based on the number of associations that occurred during the last T time-slots and are given as follow for all $k \in \mathcal{K}$:

$$w_k(t) \stackrel{\text{def}}{=} \frac{1}{1 + \sum_{n \in \mathcal{N}} \sum_{\tau=t-T+1}^t x_{kn}(\tau)}, \quad (12)$$

where $x_{kn}(\tau) = 1$, if k is associated to n at time τ and $x_{kn}(\tau) = 0$ otherwise. For simplification, we omit the variable

(t) from the weights when there is no possible confusion. The vector \mathbf{w} denotes the weights vector and is given by $\mathbf{w} = [\underbrace{w_1, \dots, w_1}_{N \text{ elements}} \cdots \underbrace{w_K, \dots, w_K}_{N \text{ elements}}]^T$. Similar procedure is followed in order to calculate the weights to ensure fairness between the SBSs at time t . Hence, for all $n \in \mathcal{N}$:

$$w_n(t) \stackrel{\text{def}}{=} \frac{1}{1 + \sum_{k \in \mathcal{K}} \sum_{\tau=t-T+1}^t x_{kn}(\tau)}, \quad (13)$$

and the corresponding weights vector \mathbf{w} is given by $\mathbf{w} = [\underbrace{w_1, \dots, w_N}_{\text{user 1}} \cdots \underbrace{w_1, \dots, w_N}_{\text{user K}}]^T$.

IV. NP-HARDNESS

This section proves the NP-hardness of the unweighted user-BS association problem (10) by reducing a known NP-complete problem to problem (10) in polynomial time. In this paper, the MAX ONES problem [24] is reduced to problem (10). The NP-hardness proof is divided into two parts. First, Lemma 1 proves the NP-hardness of a sub-problem of MAX ONES called 0-VALID MAX ONES using a reduction from the well known NP-complete problem SET COVER. Second, Theorem 1 reduces 0-VALID MAX ONES to problem (10).

Please note that the NP-hardness of the weighted user-BS association does not imply the NP-hardness of the unweighted user-BS association [19]. For instance, the weighted 0-1 knapsack problem is NP-hard whereas the unweighted case is polynomial time solvable. Also, the weighted sum of completion times on single machine with preemptive scheduling is NP-hard whereas the unweighted case is polynomial-time solvable [25]. Moreover, the structure of the matrix \mathbf{A} cannot make the unweighted problem (10) easy to solve because \mathbf{A} is a real-valued matrix and is not likely to be uni-modular.

The NP-hardness of the user-BS association problem comes from two main facts: (i) the objective function which tries to pack as much SUs as possible in the solution, i.e., maximize the number of associated SUs and (ii) the SINR constraints which creates conflicts as the SUs are associated. To clarify the ambiguity that may occur between the user-BS association problem and the one-to-one matching problem (which is an easy one), we briefly discuss the main differences.

The one-to-one matching problem includes the maximum matching in bipartite graph, the assignment problem and the stable marriage problem. In the maximum matching problem for example, there are no conflicts between the selected edges that do not share a common vertex while every SU-BS pair associated creates conflicts to the whole network. Further, the maximum matching problem was proven to be NP-hard when there are a certain pair of edges that cannot exist simultaneously in a feasible solution [26]. Note also that, some variants of the assignment problem and the stable marriage problem, roughly when the assignment includes coupled constraints as in the SINR constraints, are shown to be NP-hard [27], [28].

In what follow, the symbols \wedge (or \vee), \bigvee (or \bigwedge), and \neg denote the logical operators: disjunction, conjunction, and

negation, respectively. The notation $P_1 \propto P_2$ denotes that problem P_1 is reducible in polynomial time to problem P_2 .

Definition 1 (A binary constraint [24]):

A binary constraint is a function $f : \{0, 1\}^k \rightarrow \{0, 1\}$ for some $k \in \mathbb{N}$. We say that a binary constraint f is satisfied by an input $\mathbf{s} \in \{0, 1\}^k$ if $f(\mathbf{s}) = 1$.

Definition 2 (A 0-valid binary constraint [24]):

A binary constraint f is 0-valid if $\mathbf{s} = \mathbf{0}$ and $f(\mathbf{s}) = 1$.

Definition 3 (0-VALID MAX ONES problem [24]):

INSTANCE: A 0-valid binary constraint $f(x_1, \dots, x_n)$ of n Boolean variables x_1, x_2, \dots, x_n .

OBJECTIVE: Decide if there are assignments to x_1, x_2, \dots, x_n that satisfy $f(\cdot)$ and find the one which has the most number of true variables, that is to say $\max \{\sum_i x_i\}$.

Definition 4 (SET COVER problem, NP-complete [29]):

INSTANCE: A set of m elements called the universe. A finite family \mathcal{J} of finite sets S_j where $\mathcal{J} = \{\{S_j\} \forall j\}$, and a positive integer k .

OBJECTIVE: Decide if there is a subfamily $\{T_h\} \subseteq \mathcal{J}$ that contains $e \leq k$ sets such that $\bigcup_h T_h = \mathcal{U}$.

Without loss of generality, an instance of 0-VALID MAX ONES problem is given by:

$$\underbrace{(\bigvee_{i \in S_1} \neg x_i)}_{\text{clause 1}} \wedge \underbrace{(\bigvee_{i \in S_2} \neg x_i)}_{\text{clause 2}} \wedge \cdots \wedge \underbrace{(\bigvee_{i \in S_L} \neg x_i)}_{\text{clause L}} = \bigwedge_{l \in L} \bigvee_{i \in S_l} \neg x_i, \quad (14)$$

where S_l for all $l \in L$, is a subset of $\{1, 2, \dots, n\}$. Equation (14) is the conjunction of disjunctions of L clauses on the negated variables $\neg x_1, \dots, \neg x_{|S_l|}$.

Lemma 1 The 0-VALID MAX ONES problem is NP-hard.

Proof: See Appendix A. ■

Theorem 1 The unweighted user-BS association problem (10) is NP-hard.

Proof: See Appendix B. ■

The proof of Theorem 1 is useful in wireless networks where the user-BS association problem (10) is often encountered. Unfortunately, due to Theorem 1, solving this problem optimally requires a BF method and needs vast computational capabilities unless $P = NP$. The motivation behind the proof of Theorem 1 is to find good algorithms that are less complex and perform close to the optimal solution.

The next two sections present the optimal solution along with the proposed algorithms for the weighted and unweighted user-BS association problems.

V. OPTIMAL SOLUTIONS

This section derives the optimal solutions for problems (10) and (11). The optimal solution can be calculated by two approaches, namely, the BF algorithm and the B&B CPLEX-based algorithm. The BF algorithm is based on the enumeration of all possible associations and picking the one with the best value. On the other hand, the B&B CPLEX-based

algorithm reduces the search space, and hence the complexity, compared to the BF algorithm using the branching and the bounding approaches. These techniques are used as a reference for comparison against proposed algorithms.

In what follows, the complexity of the BF algorithm is derived for the unweighted user-BS association problem (denoted UBF) and for the weighted user-BS association problem (denoted WBF).

A. Unweighted User-BS Association

The basic steps of the UBF algorithm are the generation of all possible associations which are given by the enumeration of all combinations given by $C(K, N)$:

$$C(K, N) = \sum_{n=1}^{X_{(1)}} n! \binom{X_{(1)}}{n} \binom{X_{(2)}}{n}, \quad (15)$$

where $\binom{\cdot}{\cdot}$ denotes the binomial coefficient and $X_{(1)} \stackrel{\text{def}}{=} \min(K, N)$ and $X_{(2)} \stackrel{\text{def}}{=} \max(K, N)$.

Without loss of generality, let $N < K$, then:

$$C(K, N) \leq \sum_{n=1}^N N! \binom{K}{N} \binom{N}{N}. \quad (16)$$

From [30], an upper bound of the binomial coefficient is given by $\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \frac{n^k}{k!}$. Therefore:

$$C(K, N) \leq \begin{cases} K \cdot N^K & \text{if } N > K, \\ N \cdot K^N & \text{if } N < K. \end{cases}$$

The complexity of the UBF algorithm is denoted by UBF-C . Besides the enumeration of all possible combinations, the UBF algorithm runs through all the constraints, which is a matrix multiplication and has a complexity of $O(p \cdot q)$, equivalently $O(K^2 \cdot N^2)$. Therefore, $\text{UOPT-C} \in O(K^2 \cdot N^2 \cdot C(K, N)) \in \mathcal{X}_1$ where \mathcal{X}_1 is given by:

$$\mathcal{X}_1 \stackrel{\text{def}}{=} \begin{cases} O(K^3 \cdot N^{K+2}) & \text{if } N > K, \\ O(N^3 \cdot K^{N+2}) & \text{if } N < K, \\ O(N^5 \cdot N!) & \text{if } N = K. \end{cases} \quad (17)$$

B. Weighted User-BS Association

The WBF algorithm follows mainly the same principle as of the UBF algorithm with a slight difference. After the generation of all combinations, each step calculates the weights (for a fixed t) for those combinations that satisfy the constraints and picks the one with the maximum value. The constraints verification requires $O(p \cdot q)$, equivalently $O(K^2 \cdot N^2)$ and the calculation of the weights of those solutions requires $O(q)$, equivalently $O(K \cdot N)$, which gives a complexity of $O(K^3 \cdot N^3)$. Therefore, the complexity of the WBF algorithm, denoted by WBF-C , is $\text{WBF-C} \in O(K^3 \cdot N^3 \cdot C(K, N)) \in \mathcal{X}_2$ where \mathcal{X}_2 is given by:

$$\mathcal{X}_2 \stackrel{\text{def}}{=} \begin{cases} O(K^4 \cdot N^{K+3}) & \text{if } N > K, \\ O(N^4 \cdot K^{N+3}) & \text{if } N < K, \\ O(N^7 \cdot N!) & \text{if } N = K. \end{cases} \quad (18)$$

C. Branch-and-Bound CPLEX-based Solution

The B&B method is a well known method to solve discrete and combinatorial optimization problems [22]. It enumerates all possible solutions in a rooted tree. Then, it explores the branches of the rooted tree and estimates upper and lower bounds on the optimal solution.

In this paper, the B&B algorithm with the CPLEX solver [31], denoted by B&B CPLEX-based algorithm, is used to calculate the optimal solutions of problems (10) and (11).

In what follows, we examine the time complexity of the B&B CPLEX-based algorithm. This algorithm uses the branch-and-cut (B&C) method to solve the linear integer program [31]. More specifically, B&C calls the B&B and the cutting planes methods to solve the problem. Note that, at each step, the algorithm relaxes the linear integer problem to a continuous linear problem. In this relaxation step, the simplex algorithm is used to solve the relaxed problem. Any other linear programming-based algorithm can be used instead of the simplex algorithm.

As for the worst case complexity, the simplex algorithm has an exponential-time complexity [22]. Further, the B&B and the cutting planes methods used in the algorithm, have to visit all the nodes of the binary search tree which makes the complexity exponential. As for the best case, the algorithm needs to solve the linear programming relaxation only once, i.e., it finds the binary variables by the simplex algorithm. Hence, the best case time complexity depends on the best case time complexity of the simplex algorithm. Since the simplex algorithm has polynomial-time complexity on average [22], then the best case time complexity of the B&B CPLEX-based algorithm is upper bounded by a polynomial.

To summarize, the complexity of the B&B CPLEX-based algorithm is:

- exponential in the worst case; and
- polynomial in the average and best cases.

In practice, the B&B CPLEX-based algorithm works well as experiments show [31]. To have an idea about the difference between the running time of the BF algorithm and the B&B CPLEX-based algorithm, let us see an illustrative example. If the input is fixed to $K = 10$, $N = 6$ and the matrix \mathbf{A} is a priori known then, based on a computer characterized by “Intel(R) Core(TM) i7-3770 CPU @ 3.40 GHz 3.40 GHz”, the running time for the BF algorithm is approximately equals to 4 seconds whereas it is approximately equals to 0.1 seconds for the B&B CPLEX-based algorithm.

VI. HEURISTIC SOLUTIONS

This section describes the proposed algorithms to solve both problems (10) and (11), which consist of simple but efficient greedy algorithms.

We define $\mathbf{G} = [g_{kn}]$ for all $k \in \mathcal{K} \cup \{0\}$ and for all $n \in \mathcal{N} \cup \{0\}$ to represents the matrix of channel gains. In the pseudo-codes of the algorithms, we use the following notation $x, y \leftarrow z, t$ to assign z to x and t to y .

A. Unweighted Maximum Relative Channel Gain (UMRCG)

The proposed algorithm to solve the unweighted problem (10) is denoted by UMRCG and is given in Algorithm 1. It solves the unweighted user-BS association problem heuristically based on a greedy method.

Algorithm 1: UMRCG

Input: Network parameters: $\mathbf{G}, K, N, \gamma, \beta, \gamma_0, \beta_0$
Output: A near optimal solution: \mathbf{a}

```

1 Create the matrix  $\mathcal{U}$  according to (19)
2  $\mathbf{a}, \mathbf{p} \leftarrow [], 0$ 
3 while  $\mathbf{p} < K \cdot N$  do
4    $k, n \leftarrow \max(\mathcal{U})$   $\mathbf{a}[n] \leftarrow k$   $\text{sinr}, \text{sinr}_0 \leftarrow \text{SINR}(\mathbf{a})$   $\text{bool} \leftarrow \text{false}$ 
5   for  $j = 1$  to  $\text{length}(\text{sinr})$  do
6     if  $\text{sinr}[j] \geq \beta$  and  $\text{sinr}_0 \geq \beta_0$  then
7        $\text{bool} \leftarrow \text{true}$ 
8     else
9        $\mathbf{a}[n], \text{bool} \leftarrow [], \text{false}$  break
10  if  $\text{bool}$  is true then
11     $\text{eliminate}(k, n)$ 
12   $\mathbf{p} \leftarrow \mathbf{p} + 1$ 
13 return  $\mathbf{a}$ 

```

First, in line 1, it creates a matrix $\mathcal{U} = [u_{kn}]$ for all k and n as follows:

$$u_{kn} = \frac{g_{kn}}{\sum_{k' \neq k} g_{k'n}}. \quad (19)$$

Note that this matrix plays a key role in the proposed algorithm. In fact, u_{kn} represents the receivable signal power of k divided by the sum of receivable signal powers of other $k' \neq k$. Hence, u_{kn} can be seen as the inverse of the price of associating k to n .

After the creation of the matrix \mathcal{U} , Algorithm 1, in line 2, initializes the association vector \mathbf{a} to the empty vector and the counter \mathbf{p} to zero. The association vector defines the choice of each $n \in \mathcal{N}$, i.e., $\mathbf{a}[n] = k$ means that k is associated to n . Next, line 3 traverses the whole matrix \mathcal{U} inside the while loop. At every iteration in this loop, Algorithm 1, in line 4 finds, using the function $\min(\cdot)$, the indexes k and n of the largest element of \mathcal{U} . Then, the algorithm associates k to n . According to the association vector \mathbf{a} created so far, the algorithm calculates the SINRs, using the function $\text{SINR}(\cdot)$, of the SUs and of the MU. For every calculated SINR, the algorithm tests whether it is greater or equal than the thresholds β and β_0 as given in lines 5 and 6. If the association vector does not violate any SINR constraint so far, a Boolean variable `bool` is assigned a true value. If not,

k is dissociated from n , `bool` is set to false and the loop is broken. In line 10, if `bool` is true, which means that the newly association $\mathbf{a}[n] = k$ is valid for all associated pairs SU-SBS, then the corresponding k and n cannot be used for any further association in the subsequent iterations. Therefore, the function $\text{eliminate}(\cdot, \cdot)$, in line 11, sets the row k and the column n of \mathcal{U} to a very large number to prevent choosing them in next iterations. Note that this guarantees that constraints (3b) and (3c) are not violated. In line 12, the counter \mathbf{p} is updated and the while loop continues. Finally, when all the elements of the matrix \mathcal{U} are evaluated, the algorithm halts and returns a sub-optimal user-BS association vector \mathbf{a} .

The UMRCG algorithm runs in polynomial time. The creation of the matrix \mathcal{U} requires $O(K \cdot N)$ if we store the sum $s_n = \sum_{k=1}^K g_{kn}$ in a list of N elements and we calculate u_{kn} as $u_{kn} = \frac{g_{kn}}{s_n - g_{kn}}$ for all k and n . The while loop requires $O(K \cdot N)$ in the worst case. The function $\min(\cdot)$ requires $O(K \cdot N)$. The SINR function needs to calculate the SINR of every associated pairs SU-SBS and of the pair MU-MBS which requires $O(K \cdot N)$ by the same technique used to create the matrix \mathcal{U} . Line 5 through line 9 require $O(N)$ in the worst case. At the end, the function $\text{eliminate}(\cdot, \cdot)$ goes through the row k and the column n which requires $O(K + N)$. Finally the overall complexity of the UMRCG algorithm, denoted by UMRCG-C, is given in the worst case by $O(K \cdot N + 2 \cdot K^2 \cdot N^2 + K^2 \cdot N + 2 \cdot K \cdot N^2)$, which can be simplified to:

$$\text{UMRCG-C} \in O(K^2 \cdot N^2). \quad (20)$$

B. Weighted Maximum Relative Channel Gain (WMRCG)

The proposed algorithm to solve the weighted problem is denoted by WMRCG. It is divided into two steps. The first step, in line 1, is the calculation of the weights according to (12) or (13) during the window T . The second step, from line 2 to line 20, the algorithm WMRCG solves the weighted user-BS association problem using a procedure similar to the one described in the UMRCG algorithm. The main differences between the UMRCG algorithm and the WMRCG algorithm are the criterion in line 3 and the update of the weights in lines 19 and 20. In line 3, the algorithm WMRCG creates the matrix $\mathcal{W} = [w_{kn}]$ of SU-SBS pairs for all k and n as follows:

$$w_{kn} = \frac{w_{k|n} g_{kn}}{\sum_{k' \neq k} g_{k'n}}, \quad (21)$$

where $w_{k|n}$ is w_k or w_n , depending on whether to balance the load among the SBSs or to be fair between the SUs as discussed previously.

Likewise, the WMRCG algorithm runs in polynomial time for a fixed period t . On the one hand, the first step of calculating the weights needs to go through the association vector during the window of T time-slots and calculates how many times k (resp. n) has been associated according to (12) (resp. (13)). This requires $O(K \cdot N \cdot T)$. On the other hand, similarly to the previous analysis of the UMRCG algorithm,

Algorithm 2: WMRCG

Input: Network parameters: $T, \mathbf{G}, K, N, \gamma, \beta, \gamma_0, \beta_0$
Output: A near optimal solution \mathbf{a}

```

1 Calculate the weights in a period of size  $T$ 
2 for  $t \geq T$  do
3   Create the matrix  $\mathcal{W}$  according to (21)
4    $\mathbf{a}, \mathbf{p} \leftarrow [], 0$ 
5   while  $\mathbf{p} < K \cdot N$  do
6      $k, n \leftarrow \max(\mathcal{W})$ 
7      $\mathbf{a}[n] \leftarrow k$ 
8      $\text{sinr}, \text{sinr}_0 \leftarrow \text{SINR}(\mathbf{a})$ 
9      $\text{bool} \leftarrow \text{false}$ 
10    for  $j = 1$  to  $\text{length}(\text{sinr})$  do
11      if  $\text{sinr}[j] \geq \beta$  and  $\text{sinr}_0 \geq \beta_0$  then
12         $\text{bool} \leftarrow \text{true}$ 
13      else
14         $\mathbf{a}[n], \text{bool} \leftarrow [], \text{false}$ 
15        break
16    if  $\text{bool}$  is true then
17       $\text{eliminate}(k, n)$ 
18     $\mathbf{p} \leftarrow \mathbf{p} + 1$ 
19  Move the window  $T$ 
20  Update the weights according to (12) or (13)
21 return  $\mathbf{a}$ 

```

the complexity of the second step of the WMRCG requires $O(K^2 \cdot N^2)$. Finally, the overall complexity of the WMRCG algorithm, denoted by WMRCG-C, is given by:

$$\text{WMRCG-C} \in O(N \cdot K \cdot T + K^2 \cdot N^2). \quad (22)$$

The complexity of the different algorithms is summarized in Table I.

Algorithm	Complexity, $K < N$	Example
UBF	$O(K^3 \cdot N^{K+2})$	$4 \cdot 10^{18}$
WBF	$O(K^4 \cdot N^{K+3})$	$8 \cdot 10^{20}$
UMRCG	$O(N^2 \cdot K^2)$	$4 \cdot 10^4$
WMACG	$O(N \cdot K \cdot T + K^2 \cdot N^2)$	$24 \cdot 10^4$

TABLE I
COMPLEXITY OF THE ALGORITHMS

We see that UMRCG and WMRCG have very low complexity compared to the UBF and WBF. In fact the complexity of both algorithms is quadratic in either K or N . Notice that UMRCG and WMRCG have almost the same order of complexity unless T is of the same order as K^2 and N^2 . As an illustrative example, we set $K = 10$, $N = 20$, and $T = 1000$ in the third column of table I. We see the huge difference in the computational complexity between $4 \cdot 10^{18}$ of the UBF algorithm and $4 \cdot 10^4$ of the UMRCG algorithm. These results demonstrate the advantage of using heuristic algorithms and show how the proposed algorithms are computationally simple.

Since both the weighted and unweighted problems are NP-hard, there are no polynomial time algorithms that solve them

optimally unless $P = NP$. Therefore, our proposed algorithms can be used and implemented to solve such problems in real scenarios. Note, however, that the proposed algorithms do not approximate the optimal solution theoretically, i.e., we cannot argue that, for any instance of size ℓ of both problems, the ratio between the solutions of the proposed algorithms and the optimal algorithm is at least $\rho(\ell) < 1$. A rigorous analysis of the performance ratio of the greedy algorithms against the optimal would be an extensive work that cannot be integrated with this work. In fact, one has to study the hardness of approximation of the user-BS association problem first in order to guarantee the existence of an approximation algorithm. In [32], the authors proved that it is NP-hard to approximate the one-slot scheduling problem under the abstract SINR constraints (which is very similar to the unweighted user-BS association problem) to within $n^{1-\epsilon}$, for any $\epsilon > 0$. Therefore, the user-BS association problem is apparently hard to approximate. Such contribution is left for future work.

VII. SIMULATION RESULTS

In this section, the performance of the proposed algorithms is demonstrated by simulations. It is assumed that the path loss coefficient is $\alpha = 4$ which is a typical value in cellular networks [20], and the radius of the circle where the SBSs are located is $R = 20$ m [3]. Unless otherwise specified, the transmit SNR of the MBS and of the SBSs are set to $\gamma_0 = 40$ dB and $\gamma = 20$ dB, respectively. The SINR thresholds used for the MU and for the SUs are given respectively by $\beta_0 = 0$ dB and $\beta = 1$ dB, and the number of SUs is $K = 10$. The WMRCG is executed with a window of size $T = 50$. The B&B CPLEX-based algorithm is implemented using the OPTI Toolbox [33] under MATLAB using the IBM ILOG CPLEX solver [31].

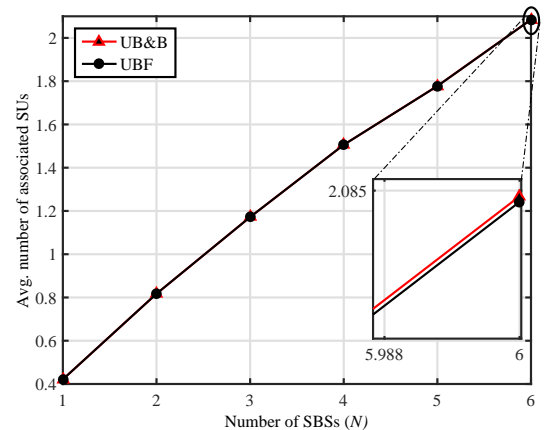


Fig. 2. Performance of unweighted optimal solutions.

Fig. 2 compares UBF algorithm and B&B CPLEX-based algorithm for the unweighted problem (denoted UB&B). We see that UBF slightly outperforms UB&B especially when N is high. When $N = 6$, UBF solution is .009% far away from UB&B one. However, this small difference is generally due to the floating points representation errors of B&B CPLEX-based algorithm. Fig. 2 along with the complexity analysis in Table I

illustrate that UB&B algorithm allows us to obtain tight-to-optimal performance with relatively low computational complexity. This motivates us to use B&B CPLEX-based algorithm in our next simulations.

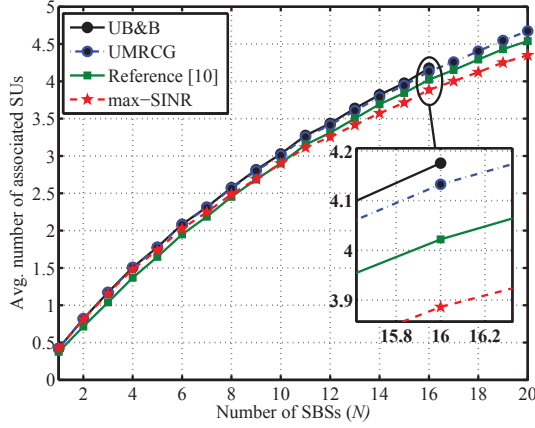


Fig. 3. Performance of UB&B and UMRCG with comparison to max-SINR and reference [10] for the unweighted user-BS association problem.

Fig. 3 shows the average number of associated SUs for the unweighted user-BS association problem (10). It compares UB&B, UMRCG, a benchmark algorithm denoted by max-SINR and a recently proposed algorithm [10]. (The algorithm in [10] is adapted to our situation.) In the max-SINR algorithm, each SU is associated to the strongest SBS signal it receives whereas in the criterion used in [10] each SU is associated to an SBS according to the sum of the received interference. This criterion works well for [10] since all the SBSs are associated in the end and therefore the sum of the received interference is perfectly estimated. We see that UMRCG algorithm has very close performance to the optimal solution. E.g., UMRCG solution is .958% far away from UB&B solution when $N = 16$. Furthermore, the proposed UMRCG algorithm outperforms max-SINR algorithm since the latter does not provide a good interference management among the BSs. Moreover, our proposed algorithm beats the algorithm in [10] since in our proposed algorithm, some of the SBSs may not be associated and therefore the amount of interference is overestimated. Note that, the performance of proposed algorithms depend on the number of SUs K and SBSs N (as shown in Fig. 3), on the transmit SNR and on the SINR thresholds. Next, we demonstrate the effect of the transmit powers and the thresholds on the performance of the proposed solutions.

Fig. 4 plots the average number of associated SUs versus the transmit SNR γ of the SBSs. When γ grows, the SINR received at the SUs grows and more SUs are expected to be associated which is illustrated in the first part of the x-axis in Fig. 4 when the curves increase. When γ continues to grow, the interference at the MU grows too and becomes harmful. Hence, the MU is not satisfied and the average number of SUs must decrease to respect the MU's QoS. This is illustrated in the second part of the x-axis in Fig. 4 when the curves dip. Notice that for high γ , if γ_0 increases, then the average

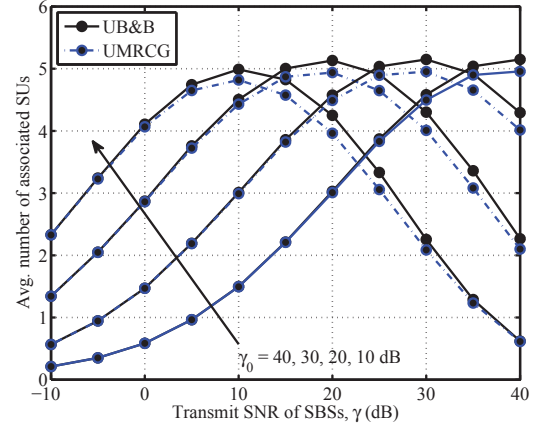


Fig. 4. Performance of UB&B and UMRCG versus the transmit SNR of the SBSs γ . $N = 10$.

number of associated SUs increases. E.g., we observe that when $\gamma = 40$ dB, the average number of associated SUs increases from .6 to approximately 5 as γ_0 increases from 10 dB to 40 dB. On the other hand, for smaller γ , if γ_0 increases, then less SUs are associated. Therefore, for a given value of the transmit SNR of the MBS, γ_0 , there is an optimum value of the transmit SNR of the SBSs, γ , to be used in order to maximize the number of associated SUs. Finally, we can see that the proposed algorithm UMRCG still gives close-to-optimal performance for different values of transmit SNR. Fig. 4 shows the worst case ratio between the UMRCG solution and the UB&B one is at most 5%.

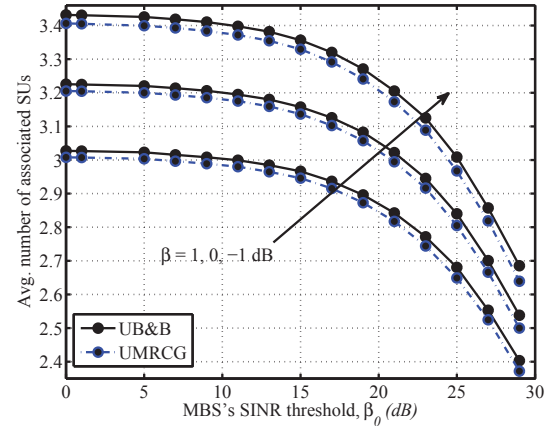


Fig. 5. Performance of UB&B and UMRCG versus the SINR thresholds of the MBS β_0 . $N = 10$.

Fig. 5 depicts the effect of SINR thresholds of SUs and of the MU. The average number of associated SUs decreases when the thresholds increase. When β_0 gets smaller, the QoS of the MU is satisfied more often and hence more SUs get associated. Furthermore, when β_0 becomes higher, the number of associated SUs decreases dramatically regardless of the value of β . It is also important to notice that the ratio between UB&B solution and UMRCG solution varies slightly as a function of β_0 and β . This ratio is still small though, which

illustrates the accuracy of the proposed heuristic solution.

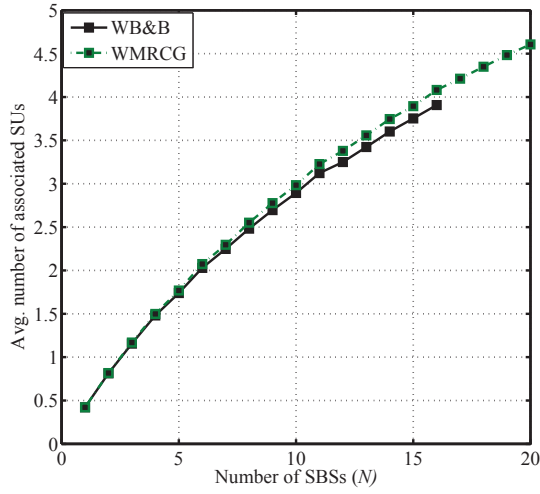


Fig. 6. Performance of WB&B and WMRCG for the weighted user-BS association.

Fig. 6 considers the proposed WMRCG solution and B&B CPLEX-based solution for the weighted user-BS association problem (11), denoted WB&B. It can be seen that WMRCG algorithm provides slightly higher number of associated SUs than WB&B algorithm since the latter does not maximize the number of associated SUs but it maximizes a weighted sum of associated SUs as can be seen by the objective function given in (11a). Comparing Fig. 3 and Fig. 6, it can be seen that the weighted solution has less performance than the unweighted one in terms of average number of associated SUs. This performance loss is compensated by gains in fairness as shown in the next simulations.

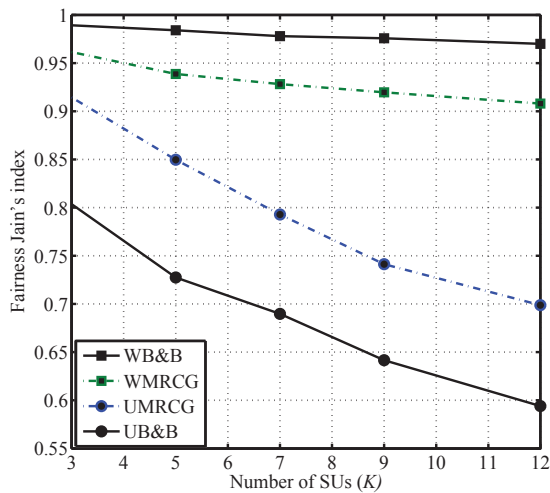


Fig. 7. Performance of the algorithms in terms of fairness between SUs. $N = 6$.

To measure the fairness, the location of SBSs is assumed fixed whereas the SUs are located randomly with uniform distribution in the network. The fairness measure used in the

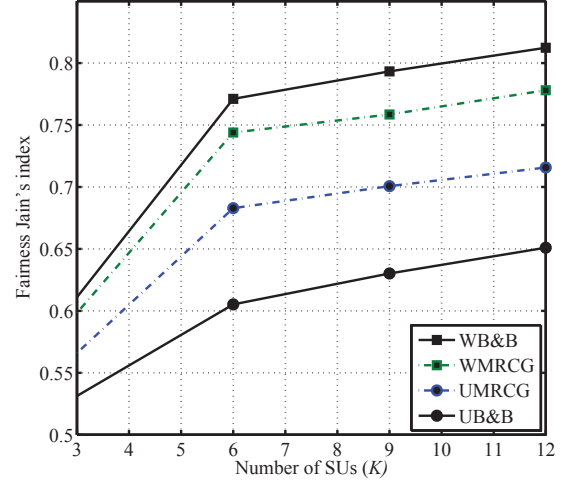


Fig. 8. Performance of the algorithms in terms of load balancing of SBSs. $N = 6$.

simulations is the well-known Jain's index [34].

Fig. 7 demonstrates the fairness of the proposed algorithms along with the optimal ones. The weights are obtained by equation (12). We observe that WB&B gives the highest fairness index. Also, WMRCG gives a high fairness index. On the other hand, UMRCG and UB&B produce the worst results of fairness index as expected. We also see that when the number of SUs increases, the network starts to densify, and the fairness of all algorithms suffer.

Fig. 8 shows the fairness between SBSs of the proposed algorithms. As discussed in Section III, the fairness between the SBSs is considered as a load balancing between the cells. The weights are obtained by equation (13). It is clear that as long as the number of SUs in the network is large, the load among different SBSs is balanced since more opportunities are given to each SBS to be associated. Further, WB&B and WMRCG still give the best results in terms of fairness between SBSs compared to UB&B and UMRCG.

VIII. CONCLUSION

This paper studies the problem of user-BS association in a HetNet of co-channel densely deployed BSs. The user-BS association problem is modeled as a linear integer program. The objective is to maximize the number of associated SUs subject to QoS constraints defined by SINR. This paper proves that the unweighted user-BS association problem is NP-hard. Then, two heuristic algorithms are proposed, namely the UMRCG algorithm and the WMRCG algorithm. Next the complexity of the proposed algorithms are derived and shown to be polynomial in time. The performance of the proposed algorithms are compared against the optimal exponential-time BF and B&B CPLEX-based algorithms. Moreover the performance is also compared against the max-SINR algorithm [5], [9] and a recently proposed algorithm in [10]. The proposed algorithms outperform all previously proposed algorithms and is close to the optimal solution as demonstrated by simulations.

The future extensions of this research will propose algorithms for joint power control and user-BS association and study the effect of statistical knowledge of channel information on the performance of these algorithms. Also, we will study the hardness of approximating the user-BS association problem and develop approximation algorithms with provable guarantees. Furthermore, fully distributed algorithms will be developed to solve the user-BS association problem using game theory and machine learning.

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APPENDIX A PROOF OF LEMMA 1

Proof: We prove that SET COVER \propto 0-VALID MAX ONES. Let I_{sc} and I_{0v} be two respective instances of SET COVER problem and the 0-VALID MAX ONES problem which are given by: $I_{sc} = (\mathcal{U}, \{\{S_j\} \forall j\}, k)$, and $I_{0v} = (f(x_1, \dots, x_n))$.

From the instance I_{sc} , we construct the instance I_{0v} as follow. From each subset S_j of \mathcal{J} , the matrix $\mathbf{M} = [m_{xy}]_{x \in \{1, \dots, X\}, y \in \{1, \dots, Y\}}$, where $X = |\mathcal{J}|$ and $Y = \max \left\{ \bigcup_j S_j \right\}$ is constructed as follows:

$$m_{xy} \stackrel{\text{def}}{=} \begin{cases} y & \text{if } y \in S_x, \\ 0 & \text{otherwise.} \end{cases} \quad (23)$$

Based on the steps given by Algorithm 3 and using the matrix defined by equation (23), the instance I_{0v} is easily obtained.

Algorithm 3: SET COVER TO 0-VALID MAX ONES

Input: An instance of SET COVER $(\mathcal{U}, \mathcal{J} = \{S_j\}, k)$.

Output: An instance of 0-VALID MAX ONES.

```

1 Construct the matrix  $\mathbf{M}$  according to (23)
2 for  $j = 1$  to  $Y$  do
3    $C_j = 1$ 
4   for  $i = 1$  to  $X$  do
5     if  $m_{ij} \neq 0$  then
6        $C_j = C_j \vee \neg x_i$ 
7  $f(x_1, \dots, x_X) = C_1 \wedge C_2 \wedge \dots \wedge C_Y$ 
8 return  $f(x_1, \dots, x_X)$ .
```

Finally, if 0-VALID MAX ONES is solved with the instance I_{0v} then the optimal solution $\mathbf{x} = (x_1, \dots, x_n)$ contains the least possible number of zeros. Let I be the set of zeros in the solution \mathbf{x} . Thus, the solution of the SET COVER problem corresponds to the subfamily of sets $\mathcal{I} = \{\{S_p\}, \forall p \in I\}$. Hence, SET COVER problem is solved with the minimum number of subsets. The reduction from SET COVER to 0-VALID MAX ONES is polynomial time as shown in Algorithm 3. Therefore, SET COVER \propto 0-VALID MAX ONES which proves Lemma 1. ■

APPENDIX B PROOF OF THEOREM 1

Proof: We show that 0-VALID MAX ONES \propto problem (10). Let $I_1 = (K, N, \mathbf{A})$ be an instance of problem (10) where K is the number of SUs, N is the number of SBSs, \mathbf{A} is the matrix defined in problem (10). Let $I_2 = (f(x_1, \dots, x_n))$ be an instance of the 0-VALID MAX ONES problem.

An instance of problem (10) can be constructed by converting the set of Boolean clauses of the binary constraint $f(\cdot)$ to a system of linear inequalities. Therefore, $f(\cdot)$ is true $\Leftrightarrow \mathbf{Ax} \leq \mathbf{1}$. Hence, the problem of maximizing the number of associated SUs while the SINR requirements are met (i.e., $\mathbf{Ax} \leq \mathbf{1}$) is equivalent to the problem of maximizing the number of true literals while the Boolean formula is true (i.e., $f(\cdot)$ is true).

In order to get the instance I_1 from the instance I_2 , the following transformation is applied. First, let $S_l = \{i_1^l, \dots, i_k^l\}$ be a subset of $\{1, \dots, n\}$ for some $l \in L$ and some $k \in \{1, 2, \dots, n\}$. Then, for each clause l of $f(\cdot)$, i.e., $\bigvee_{i \in S_l} \neg x_i$, the following system of linear inequalities is given:

$$\gamma g_{i_\sigma^l i_\sigma^l} < \left(\sum_{\substack{p=1 \\ p \neq \sigma}}^k \gamma g_{i_\sigma^l i_p^l} + 1 \right) \beta, \quad \forall \sigma \in \{1, \dots, k\}.$$

Second, this system of linear inequalities is easily solved in polynomial time to get the corresponding g_{ij} since it has more many variables than equations. Let \mathcal{A} and \mathcal{B} be the sets of solutions of the g_{ij} . Then, the remainder values of g_{ij} will be set to 0, i.e., $g_{ij} = 0, \forall i \notin \mathcal{A}, \forall j \notin \mathcal{B}$. Using this transformation, we can get the matrix \mathbf{A} , K , and N where $K = N = Y$ (Y represents the number of clauses in the instance I_2 , i.e., $Y = |L|$). Therefore an instance of problem (10) is obtained. Finally, if problem (10) is solved using I_1 and let the solution vector be \mathbf{x} , then if $x_{ij} = 1 \Leftrightarrow i = j$ and the corresponding Boolean variable is set to 1. Hence, in the instance I_2 , we have $x_i = x_j = 1$ and $x_{i'} = 0, \forall i' \neq i, j$. Therefore, 0-VALID MAX ONES is solved. At last, we can verify easily in polynomial time that the constraints evaluate true is equivalent to the Boolean formula evaluates true. Therefore, problem (10) is solved if and only if 0-VALID MAX ONES is solved.

To conclude, 0-VALID MAX ONES \propto problem (10) and therefore the unweighted user-BS association problem is NP-hard. This proves Theorem 1. ■

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